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Dynamic Contests With Bankruptcy: The Despair Effect

Abstract: We analyze a two-period contest in which agents may become bankrupt at the end of the first period. A bankrupt agent is excluded from the contest in the second period of the game. We investigate the existence of a subgame perfect equilibrium in pure strategies. We distinguish between a borrowing equilibrium in which at least one agent might be bankrupted and a non borrowing equilibrium in which no agent is bankrupted. We prove that the former occurs when the agent taking loans is relatively poor and the future does not matter very much. This action represents the Despair Effect, in which severely handicapped agents take actions that jeopardize their existence in the long run but are currently helpful. We find conditions under which borrowing and non borrowing equilibria overlap and do not overlap. We provide an example in which no equilibrium exists.

Keywords: dynamic contest, bankruptcy

JEL classification: C72, D72, D74

1 Introduction

The theory of contests studies conflicts in which agents expend effort to obtain a prize. In static contests, aggregate effort is maximized when agents are identical. If I play tennis against Nadal, I will expend very little effort -because my chances of winning are very small- and Nadal will expend very little effort because he does not need much effort to defeat me. In dynamic contests, this situation translates into the *Discouragement Effect*, where lagging players expend little effort or throw in the towel -because they will lose with a high probability- and players with a large advantage expend little effort because they do not need much to win (see Konrad (2012) and references therein).

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However, in some dynamic contests, the contrary may occur. Suppose that in the Champions League, a soccer team, after losing in the first half of the first round, is offered a drug that will increase their performance in the second half, but, with some probability the drug will be discovered and the team disqualified. If the losing team is defeated by, say 1–0, it might reject the offer on the grounds that it is too risky. However, if the team is losing 3–0, it might accept the drug because the chances that it can overturn the result without extra help are slim. The result will be more effort by both teams in the second half of the game. Thus, heterogeneity in players may increase the aggregate effort, at least in some periods. We call this situation the *Despair Effect* because losers may find it optimal to take risky actions that would not be sensible if their chances were higher.

In this paper, we present a two-period complete information contest in which two agents are endowed with money and can get extra money in a capital market.¹ Potential lenders can invest either in the safe asset or in a contestant. The latter is risky because if this contestant does not win, the investors obtain no return. The capital market equalizes the expected returns of both assets. Agents with shallow pockets may overcome this handicap by raising loans and competing on more equal terms with agents with deep pockets. This is a very stylized picture of wars among empires and repeated competition among firms for procurement (aircraft in the US Navy or construction firms).²

A crucial assumption of our model is that a contestant unable to repay the loan will be excluded in the second-period contest. This assumption is an idealization of the problems faced by a country or a firm that is unable to repay its debts. It has been microfounded by Bolton and Scharfstein (1990) in a strategic finance setup and used by Eaton and Gersovitz (1981) in their analysis of sovereign debt, in which a country that refuses to repay faces an embargo on future loans. In the next section, we discuss this assumption. For the time being let us recall Mr. Micawber's famous, and often quoted, recipe for happiness (see Dickens (1850)):

Annual income twenty pounds, annual expenditure nineteen [pounds] nineteen [shillings] and six [pence], result happiness. Annual income twenty pounds, annual expenditure twenty pounds ought and six, result misery.

This is explained by the fact that, in the novel, the slightest debt will put the debtor in jail, no matter how small. Another example of our assumption is the

¹ See Brander and Lewis (1988) for a study of the role of financial constraints and bankruptcy in a duopoly. Gale and Che (1997) also consider budget constrained contestants, but they do not consider bankruptcy.

² The initial motivation for this research came from a statement of Sir Norman Foster in the film "How much does your building weigh, Mr. Foster?" About the dire straits suffered by his studio before they were awarded the HSBC Hong Kong building.

recent FIFA proposal to enforce a budget balance among all of the clubs that play European competitions. To play, clubs must prove that they have no outstanding payments to players, to each other or to the tax authorities (Franck 2014).

We distinguish between two scenarios. In the first, one of the agents (the rich agent) has a very large money endowments, so he never takes a loan and never faces the risk of liquidation. The other agent (the poor agent) has a limited money endowment, he might get a loan, so he either wins the contest or faces liquidation. We call this scenario Rich Man-Poor Man. We prove that the pure strategy subgame perfect Nash equilibrium (SPNE) of this game is unique and can be of two, mutually exclusive, types:³

1. Non borrowing equilibrium: The poor agent finances his expenses entirely with his endowments.
2. Borrowing equilibrium: The poor agent finances part of his expenses in the capital market.

In the non borrowing equilibrium both agents spend less than in the standard one shot Nash equilibrium (NE). In the borrowing equilibrium, both agents spend more than in the one shot NE. The latter is an example of the despair effect at work. It also has predatory features because the rich agent spends a large quantity of money that drags the poor agent into borrowing, which, given the risk of bankruptcy for the latter, increases the expected prize received by the rich agent in the second period. As a consequence, the extent of rent dissipation depends on which equilibrium occurs and might exceed or fall short of that in the standard one shot NE.

The (non) borrowing equilibrium exists when the poor agent is (resp., is not) very poor and does not care (resp., does care) much about the future. The role of the discount is clear: if an agent does not care much about the future, the risk of bankruptcy has small payoff consequences, so he is inclined to obtain a loan. The role of the endowment is where the Despair Effect comes into the picture. A loan allows the poor agent to compete on equal footing with the rich agent. When both agents have similar monetary endowments, a loan does not mean much to the poor agent, in terms of helping him to compete in the first period, and it brings a risky outcome.⁴ We also show that for intermediate values of the poor agent's endowment and the discount rate, a SPNE may not exist.

³ For tractability reasons, the analysis in this paper is restricted to the case of pure strategies.

⁴ Our results can be interpreted in terms of endogenous time preference (Becker and Mulligan 1997). In our case, the rich man is an endogenously more patient player, because he runs a smaller risk of bankruptcy, thereby having a better prospect of surviving until the second period to claim the prize. Therefore bankruptcy adds to the standard determinant of time preference like wealth, mortality, addiction and uncertainty.

In the second scenario, both agents are identical, have limited endowments, and may access the capital market. We call this scenario Poor Man-Poor Man. This scenario is meant to capture the polar situation to the previous section and to see the impact of deep pockets on the equilibrium outcome. Now we have two equilibria: one in which neither agent borrows and another in which both agents borrow. We show that these equilibria have properties that match those in the Rich Man-Poor Man scenario except that here both equilibria can coexist. The borrowing equilibrium is an example of the despair effect affecting both players. In this case we have double predation, where both contestants go to the end of their tether in an effort to ruin the opponent. This shows that predation is not caused by deep pocket but by the competitive nature of our game.

Our paper is related to other papers in which the result of early rounds may encourage contestants to make greater efforts. Garfinkel and Skaperdas (2000) study the effect of war on pacification in subsequent periods. Sela (2011) considers a race in which the loser cares about the magnitude of the defeat and shows that the loser of the first battle may be encouraged to increase effort in the second battle to avoid dishonorable defeat. Möller (2012) and Beviá and Corchón (2013) consider two-period contests where prizes won in earlier periods improve the players' abilities or the probability of success in the second contest. Because winning in the first round has a positive impact on the outcome in the second round, players have an extra incentive to make an effort. Consequently, the discouragement effect holds only when the difference between players is sufficiently large. Another strand of dynamic contests studies contests with several rounds and where contestants are eliminated at each round, see the pioneering paper by Rosen (1986) and Fu and Lu (2012) for a recent entry and further references. Examples of these situations are promotions inside the firm, sports, the Oscars... The difference with our approach is that we do not require elimination. Elimination may or may not occur in our model depending on the equilibrium of the game. Thus in our model "money talks".

The Despair Effect considered in our paper differs from the literature above in that an effort today might bring disastrous consequences in the future. Examples of this effect abound in military history from the dictum "caja o faja" (coffin or belt, a military regalia only wore by marshals), which refers to the low rank officers commanding almost suicidal attacks that, in the case of success bring large promotions, to battles such as Leite Gulf in 1944, in which the Japanese navy committed almost all of their available ships to defend crucial oil supply lines to Japan. Similarly, at the end of Spain's mastery in Europe (1635), Spain engaged in "an all or nothing" war with France that

almost succeeded (Corteguera 2002, 143). Finally, the famous dictum in the “Communist Manifesto” (1848), “the proletarians have nothing to lose but their chains. They have a world to win” may be interpreted as another example. The Despair Effect also arises in long-distance running, where a well-known tactic for lagging runners is to make an extraordinary effort to catch up the leaders. Many times this large effort forces the athlete to abandon the race, but in some cases it pays off.

The rest of the paper proceeds as follows. Section 2 describes the model. The Rich Man-Poor Man and the Poor Man-Poor Man scenarios are analyzed in Sections 3 and 4, respectively. Section 5 closes with some final comments.

2 The Model

There are two periods and two agents, also called contestants and denoted as a and b . In each period t agents contest for a prize of value V by spending a quantity of a resource that we call money and denote by G_i^t where $i \in \{a, b\}$. In period one agent i is endowed with M_i units of money. Without loss of generality we assume that $M_a \geq M_b$. If $G_i^1 > M_i$ agent i can borrow from a credit market where money can be invested either in financing the contestants or in a riskless asset that, after a period, yields r units per unit investment. The interest rate r is determined exogenously. An investment of a unit of money in the expenses made by contestant i yields s with probability p_i (which is the probability that i wins the contest) and 0 with probability $1 - p_i$ (which is the probability that i loses the contest). Assuming that investors are risk neutral, the expected return is $p_i s$. If the capital market is competitive, we should have

$$p_i s = r. \quad [2.1]$$

Thus, if the “risky” investment is a safe deal, $p_i \cong 1$ and $s \cong r$. When the risky investment is very risky, $p_i \cong 0$ and $s \cong \infty$.

Let us now write the expected payoffs of contestant i who fully financed G_i^1 units of money through the capital market and spent them in the contest. With probability p_i it wins V but it must pay sG_i^1 . With probability $1 - p_i$, it loses and has no money to pay. Thus, the expected profits for contestant i are

$$p_i V - p_i s G_i^1 = p_i V - r G_i^1. \quad [2.2]$$

If the expenses are financed with the endowments, they have an opportunity cost of r . In any case, eq. [2.2] represents the payoff of contestant i . In the rest of the paper, without loss of generality, we set $r = 1$.

In each period there are three stages defined as follows:

1. Agents decide on the amount of expenses.
If this amount exceeds the available money, they borrow the difference, i.e. if agent i spends $G_i^t > M_i$ he can borrow $G_i^t - M_i$ in the credit market.⁵
2. The prize is awarded.
Let p_i^t be the probability that agent i obtains the prize in period t . For simplicity we will assume that the probability of winning the contest is given by a Tullock Contest Success Function (CSF).

$$p_i(G_i^t, G_j^t) = \frac{G_i^t}{G_i^t + G_j^t}, i, j \in \{a, b\}, i \neq j. \quad [2.3]$$

3. **Bankruptcy rules.** If the agent was in debt and did not win the prize, no one wants to lend him any more money, so he is excluded from the contest in period 2.

This assumption is, of course, an idealization. Couwenberg (2001) finds that the survival rate of firms after bankruptcy is 18% in the US, 20% in the UK and 6% in France. Countries may fall into several bankruptcies before they are out of the world domination game: The Spanish Habsburgs became bankrupt in 1557, 1576, 1596 and 1607 before the bankruptcies that sealed their fate in 1647 and 1653. France, bankrupt at the eve of French revolution in 1789, enjoyed an enviable position in European affairs until 1813. In both cases, however, bankruptcies had serious consequences on the role of these nations.⁶ Even near bankruptcies, such as Scotland in 1707, Great Britain in 1945, and Russia in the 1990s, paved the way for the reduced visibility of these nations in subsequent years. In fact, Paul Kennedy (1987) argues that financial overburden, caused by overexpansion in strategic commitments, is the primary cause of the decline of empires. We take this view to the limit, assuming that bankrupted nations disappear from the contest arena. Furthermore, some countries and companies preclude bankrupt agents from procurement (see Chapter 15 of US government procurement). A simple justification of this procedures is that they are a crude way of dealing with the free-rider problem that would arise if bankruptcy were followed by debt renegotiation.

⁵ We assume that an agent's borrowing decision cannot be observed by her rival. This assumption seems to us plausible and makes the model simpler than assuming otherwise.

⁶ Cruces and Trebesch (2012) construct a database of investor losses in all restructurings of sovereign debt from 1970 until 2010, covering 180 cases in 68 countries and find that "high creditor losses are associated with... longer periods of market exclusion."

We say that an agent is *active* in the second period if he can participate in the contest. If an agent loses the contest in the first period, he will be active in the second period iff $G_i^1 \leq M_i$. Note that if an agent wins the contests she can repay the loan since no maximizing agent will ever bid more than the value of the prize.

Finally, we assume that:

4. **Second period.** If there is only one active agent in this period, this agent wins the prize at no cost.⁷ If there are no active agents, the prize is not awarded. If two agents are active, they compete as in the first period. If an agent cannot repay the loan, this has no consequences because the world ends in this period. Therefore, in the second period, if both agents are active, they spend equal amounts and obtain the same payoff, which we denote by $\bar{\pi}$.

In period t , the expected payoff of agent i is

$$\pi_i^t = p_i(G_i^t, G_j^t)V - G_i^t. \quad [2.4]$$

The expected payoff for agent i for the entire game is denoted by π_i and defined as

$$\pi_i = \pi_i^1 + \delta \pi_i^2 \quad [2.5]$$

where $\delta \in (0, 1]$ is the discount rate, common to both agents.

Our equilibrium concept is subgame perfect Nash equilibrium (SPNE). Given this, all the relevant action occurs in period one. Consequently, we focus our analysis on the variables in this period and we drop the superindexes from now on. Thus $G_i^1 = G_i$, etc.

In the next section, we focus on the case in which agent a can pay out any conceivable expense from his endowments, but agent b cannot. We will call this case “Rich Man-Poor Man”. The case in which both agents are constrained (“Poor Man-Poor Man”) is analyzed in the subsequent section.

3 Rich Man-Poor Man Scenario

In this section, we assume that agent a -the rich agent- has a very large quantity of money so that he will never be constrained, and agent b -the poor agent- does not.

⁷ In the final section we discuss the possibility that a bankrupted agent is replaced in the second period by a fresh agent.

Consider a game in which the payoff functions are π_a and π_b (see 2.4) and there are no financial constraints. We call the Nash equilibrium of this game the one shot Nash equilibrium. Under our assumptions, this equilibrium is given by

$$\bar{G}_a = \bar{G}_b = \bar{\pi} = \frac{V}{4} \quad [3.1]$$

where \bar{G} is the expense of an agent in the one shot Nash equilibrium and $\bar{\pi}$ is the one shot Nash equilibrium payoffs.

We assume that the poor agent cannot finance \bar{G} out of his pocket, i.e. $M_b < \bar{G}$. Consequently, the poor agent must decide whether he wants to borrow. This case corresponds to the “deep pocket” case, which has been considered in oligopolistic markets; see Bolton and Scharfstein (1990) and the references therein. It has been argued that deep pockets yield predation. We consider the validity of such a conclusion in our set up.

There are two possible types of equilibria of the dynamic game: those in which the poor agent does not borrow –equilibrium without borrowing– and those in which the poor agent does borrow –equilibrium with borrowing. We start with the case in which the poor agent does not borrow in equilibrium.

Note first that if agent b is constrained, $M_b < V/4$. To show the existence of a non borrowing equilibrium, we must show that $G_b = M_b$ and the best reply to this expense by agent a , $G_a = \sqrt{VM_b} - M_b$, is an equilibrium. The payoff of the poor agent if he does not borrow is:

$$\pi_b^{NB} = \frac{M_b}{M_b + G_a} V - M_b + \delta \frac{V}{4} = \sqrt{M_b} V - M_b + \delta \frac{V}{4} = G_a + \delta \frac{V}{4}. \quad [3.2]$$

If he deviates and decides to borrow, he will risk bankruptcy but will increase his probability of winning in the first period. Then, his continuation payoff is

$$\pi_b^B = p_b V - \hat{G}_b + \delta \left(p_b \frac{V}{4} \right) = p_b \left(V + \delta \frac{V}{4} \right) - \hat{G}_b \quad [3.3]$$

$$\text{where } \hat{G}_b = \sqrt{V \left(1 + \frac{1}{4} \delta \right) G_a} - G_a, \text{ and } p_b = \frac{\hat{G}_b}{\hat{G}_b + G_a}. \quad [3.4]$$

A non borrowing equilibrium exists iff the payoffs when the poor agent borrows are smaller than the payoffs when he does not borrow. By setting $q \equiv \frac{V}{M_b}$ the necessary and sufficient condition can be written as:

$$\sqrt{q} - 1 \geq \frac{q}{4 \left(1 + \frac{1}{4} \delta \right)}. \quad [3.5]$$

Thus we have shown the following:

Proposition 1 *In the Rich Man- Poor Man scenario, an equilibrium without borrowing exists iff the following inequality holds.*

$$\sqrt{q} - 1 \geq \frac{q}{4(1 + \frac{1}{4}\delta)}.$$

In Figure 1 below the values (q, δ) that satisfy ((10)) are those in the area above the dashed line.

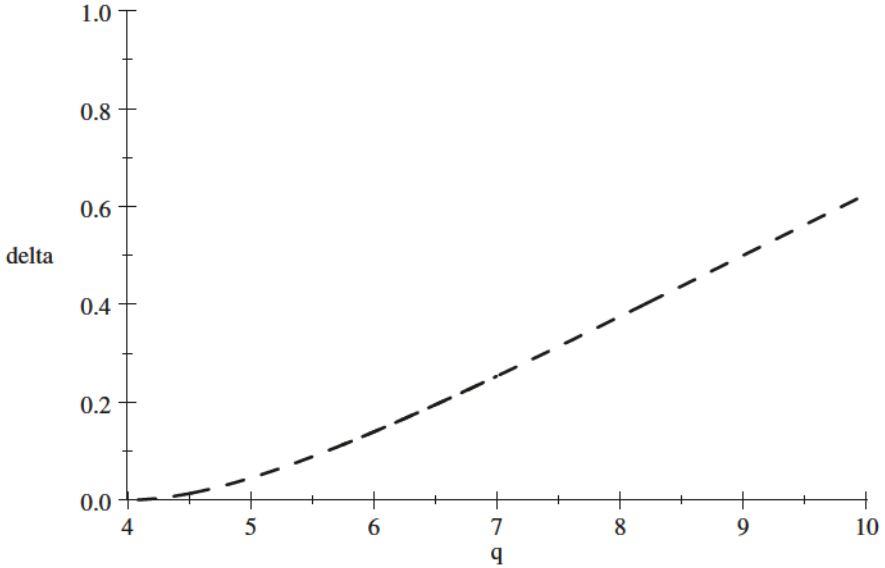


Figure 1: Non borrowing equilibrium in the rich man-poor man scenario.

Because agent b is constrained, $M_b < V/4$, that is, $q > 4$. Note the following features of this equilibrium:

1. A non borrowing equilibrium arises as a combination of patient agents and the poor agent not being very poor: If $\delta = 1$ (which is the most favorable case for the existence of a non borrowing equilibrium), eq. [3.5] implies that q must be smaller than 13.09, that is, the initial wealth of the poor agent should be at least 7% of the value of the prize.
2. In a non borrowing equilibrium, both agents spend less than if they are unconstrained. For the poor agent, this is by definition, and for the rich agent, it follows from the fact that, in equilibrium, strategies are strategic complements. Thus hard financial constraints lead *all* agents to spend less.

We now turn our attention to equilibrium with borrowing.

If an equilibrium with borrowing exists, expenses in the first period are given by:

$$G_a = \sqrt[2]{V \left(1 + \frac{3}{4}\delta\right)} G_b - G_b, \text{ and } G_b = \sqrt[2]{V \left(1 + \frac{1}{4}\delta\right)} G_a - G_a. \quad [3.6]$$

Solving eq. [3.6], we obtain

$$G_a = \frac{(1 + \frac{3}{4}\delta)^2 (1 + \frac{1}{4}\delta)}{(2 + \delta)^2} V, \text{ and } G_b = \frac{(1 + \frac{3}{4}\delta) (1 + \frac{1}{4}\delta)^2}{(2 + \delta)^2} V. \quad [3.7]$$

Both G_a and G_b are increasing in δ . Because for $\delta = 0$, $G_b = V/4 > M_b$, agent b is borrowing. The prize in case of borrowing for the rich agent is larger because with certain probability, he will be the only one surviving in the second period; thus, $G_a > G_b$. The probabilities for each player of winning the contest in the first period are:

$$p_a = \frac{(1 + \frac{3}{4}\delta)}{(2 + \delta)}, \text{ and } p_b = \frac{(1 + \frac{1}{4}\delta)}{(2 + \delta)}. \quad [3.8]$$

Thus, the payoff for the poor agent if he borrows is:

$$\pi_b^B = p_b V - G_b + \delta \left(p_b \frac{V}{4} \right) = \frac{(1 + \frac{1}{4}\delta)^3}{(2 + \delta)^2} V. \quad [3.9]$$

Let $RO_i(G_j)$ the best reply of agent i if the contest were played once and agents were not constrained by monetary endowments. For (G_a, G_b) to be an equilibrium we need to check first that $RO_b(G_a) > M_b$, because otherwise, given G_a agent b can best reply without borrowing and risking bankruptcy. Second, we must show that the poor agent does not have an incentive to deviate and play safe, that is, to not borrow and to play M_b . Because $RO_b(G_a) = \sqrt{V G_a} - G_a$, the condition $RO_b(G_a) > M_b$ can be written as:

$$V \frac{(1 + \frac{3}{4}\delta)}{(2 + \delta)} \sqrt{\left(1 + \frac{1}{4}\delta\right)} - \frac{(1 + \frac{3}{4}\delta)^2 (1 + \frac{1}{4}\delta)}{(2 + \delta)^2} V > M_b. \quad [3.10]$$

Dividing by M_b and letting $q = V/M_b$, as before, we obtain

$$q \frac{(1 + \frac{3}{4}\delta)}{(2 + \delta)} \sqrt{\left(1 + \frac{1}{4}\delta\right)} - \frac{(1 + \frac{3}{4}\delta)^2 (1 + \frac{1}{4}\delta)}{(2 + \delta)^2} > 1. \quad [3.11]$$

Given eq. [3.11], let us see that agent b does not have an incentive to deviate. If he plays M_b given that agent a is playing G_a , his payoff will be:

$$\pi_b^{NB} = \frac{M_b}{G_a + M_b} V - M_b + \delta \frac{V}{4}. \quad [3.12]$$

Thus, he will not deviate if $\pi_b^{NB} \leq \pi_b^B$ or, equivalently,

$$\frac{1}{\frac{(1+\frac{3}{4}\delta)^2(1+\frac{1}{4}\delta)}{(2+\delta)^2}q+1}q-1+\frac{\delta}{4}q \leq \frac{(1+\frac{1}{4}\delta)^3}{(2+\delta)^2}q. \quad [3.13]$$

We now prove that condition [3.13] implies condition [3.11] (as Figure 2 shows). To see this, let $F(q)$ be the function implicitly defined by

$$\frac{1}{\frac{(1+\frac{3}{4}\delta)^2(1+\frac{1}{4}\delta)}{(2+\delta)^2}q+1}q-1+\frac{\delta}{4}q = \frac{(1+\frac{1}{4}\delta)^3}{(2+\delta)^2}q, \quad [3.14]$$

and let $H(q)$ be the function implicitly defined by

$$q\left(\frac{(1+\frac{3}{4}\delta)}{(2+\delta)}\sqrt{\left(1+\frac{1}{4}\delta\right)-\frac{(1+\frac{3}{4}\delta)^2(1+\frac{1}{4}\delta)}{(2+\delta)^2}}\right)=1. \quad [3.15]$$

Recall that $q \geq 4$ and note that $F(4) = H(4) = 0$. Furthermore, there is no $q \neq 4$ such that⁸ $F(q) = H(q)$. Also note that $F(5) = 4.2986 \times 10^{-2}$ and $H(5) = 1.6048$. Thus, since both functions only cross at $q = 4$ and $F(5) < H(5)$, then $F(q) < H(q)$ for all $q > 4$, and this proves that condition [3.13] implies condition [3.11]. Summing up, we have proved:

Proposition 2 *In the Rich Man-Poor Man scenario an equilibrium with borrowing exists iff the following inequality holds.*

$$\frac{1}{\frac{(1+\frac{3}{4}\delta)^2(1+\frac{1}{4}\delta)}{(2+\delta)^2}q+1}q-1+\frac{\delta}{4}q \leq \frac{(1+\frac{1}{4}\delta)^3}{(2+\delta)^2}q.$$

Conditions [3.13] and [3.11] look quite formidable but they are easy to picture in Figure 2 below. The area below the dotted line (which is very close to the δ axis) corresponds to condition [3.11] and the area below the solid line corresponds to condition [3.13]. Thus, for any (q, δ) an equilibrium with borrowing exists in the area below the solid line.

⁸ The resolution of the system of equations using Maple given by eqs [3.14] and [3.15] and given that $q \geq 4$ and $0 \leq \delta \leq 1$ only have as a solution $q = 4$, $\delta = 0$.

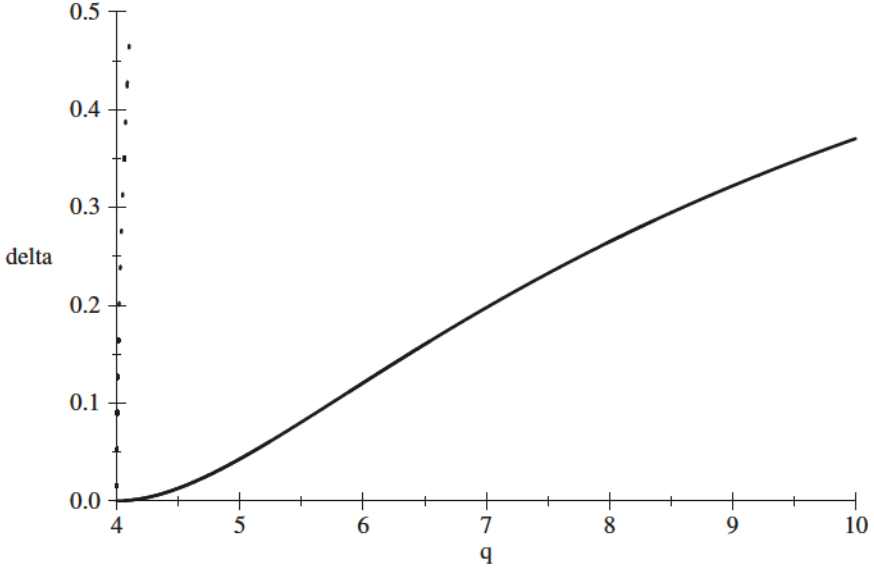


Figure 2: Borrowing equilibrium in the rich man-poor man scenario.

Condition [3.13] says that, for a given q , when the poor agent does not care much about the future (i.e., δ is close to 0), he may risk bankruptcy. Alternatively, given δ , the poorer the agent is relative to V (i.e. q is high), the more likely he is to prefer the risky strategy of borrowing resources. Thus, the borrowing equilibrium arises as a combination of impatient agents and very few endowments in hands of the poor agent. Interesting features of this equilibrium are:

1. Expenses of both players are larger than those in the one shot equilibrium. Thus, in this equilibrium, agent a challenges agent b with a large expense, and agent b accepts the challenge. This follows from the fact that the payoff of the rich agent is larger when the poor agent has a possibility of going bankrupt. Consequently, the rich agent is more aggressive in the first period. On the one hand the rich agent is affected by the “Shadow Effect” where the anticipation of a future weaker contender makes strong players more aggressive (see Brown and Milnor (2011)). On the other hand, the poor agent is affected by the Despair Effect which gives incentives to increase his expenses.
2. When q is very large (M_b is small), an equilibrium with borrowing exists iff $\delta < .876$. This implies that even under very low endowments the poor agent might not like to embark upon borrowing unless she discounts heavily the future. Thus the despair effect needs both a weak player and a relatively low importance of the future.

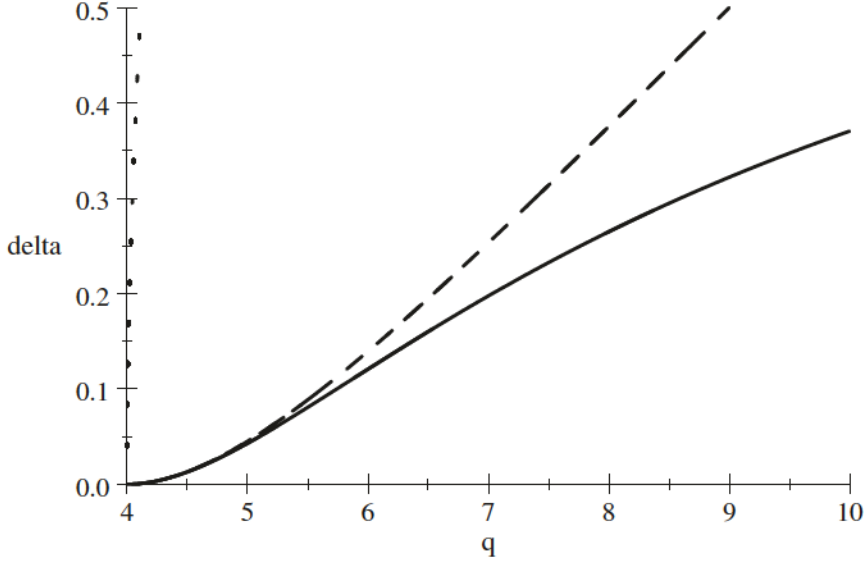


Figure 3: Equilibrium in the rich man-poor man scenario.

In Figure 3 we combine Figures 1 and 2. Note that for any pair (q, δ) between the solid line and the dashed line in Figure 3, there is no SPNE in pure strategies. Finally, note that the equilibrium, whenever exists, is unique.

To close this section we present an example of non existence of equilibrium in pure strategies.

Example 1 *Non-existence of Equilibrium in pure strategies.*

Assume that the CSF is given by eq. [2.3]. Let $V = 100$, $\delta = 0.5$, and $M_b = 10$. The best reply function for agent a is:

$$G_a = \begin{cases} \sqrt{100G_b} - G_b & \text{if } G_b \leq M_b \\ \sqrt{137.5G_b} - G_b & \text{if } G_b > M_b \end{cases}$$

The discontinuity for the rich agent is explained by the fact that for expenses of the poor larger than her endowments he risks bankruptcy so payoffs increase discontinuously for the rich. For the poor agent $G_b = \sqrt{100G_a} - G_a$ if G_a is such that $\sqrt{100G_a} - G_a \leq M_b$. That is, if $G_a \in [0, 1.27] \cup [78.73, \infty)$. For any other G_a , the poor agent has two options: either he borrows or he does not borrow. If he borrows, his expected payoff is given by:

$$\pi_b^B = p_b^1 V - \sqrt{V \left(1 + \frac{1}{4}\delta\right) G_a + G_a + \delta \left(p_b^1 \frac{V}{4}\right)}. \quad [3.16]$$

If he does not borrow, his expected payoff is given by

$$\pi_b^{NB} = \frac{M_b}{M_b + G_a} V - M_b + \delta \frac{V}{4}.$$

Therefore, he will borrow if G_a is such that $\pi_b^B > \pi_b^{NB}$ which after some calculations amounts to

$$\frac{\sqrt{112.5G_a - G_a}}{\sqrt{112.5G_a}} 112.5 - \sqrt{112.5G_a + G_a} > \frac{10}{10 + G_a} 100 + 2.5.$$

That is, he will borrow if $G_a \in [3.82, 26.18]$. Thus, the best reply of the poor agent is:

$$G_b = \begin{cases} \sqrt{100G_a - G_a} & \text{if } G_a \in [0, 1.27] \cup [78.73, \infty) \\ M_b = 10 & \text{if } G_a \in [1, 27, 3.82] \cup [26.18, 78.73] \\ \sqrt{112.5G_a - G_a} & \text{if } G_a \in (3.82, 26.18) \end{cases}$$

Thus the poor agent spends her endowments until the challenge of the rich is so large that he borrows. But if the rich spends more and more the risk of bankruptcy becomes so large that the poor decides to get rid of this risk by spending her endowments only. In Figure 4 we represent the best reply of the reach agent

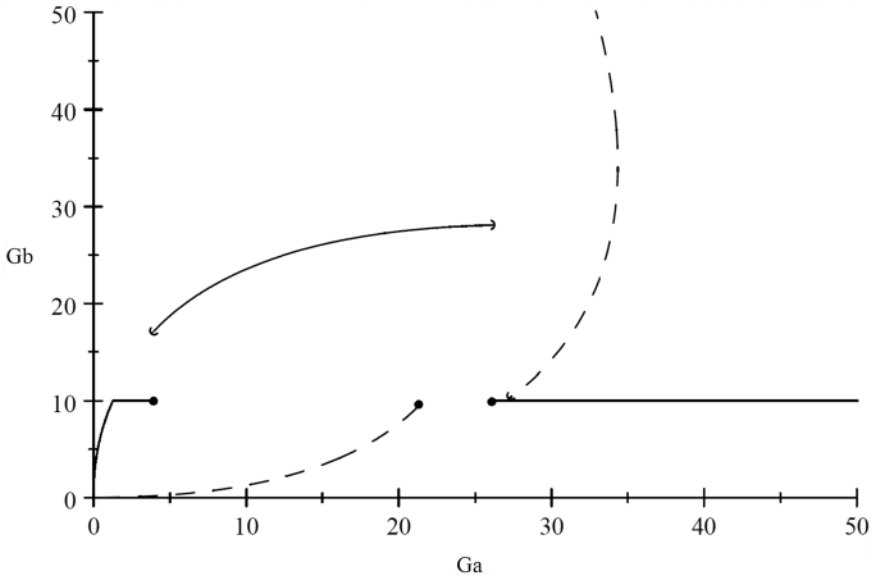


Figure 4: Non existence of equilibrium. Rich man-poor man scenario.

by a dashed line and the best reply of the poor agent by a solid line. Clearly, there is no Nash equilibrium in pure strategies.

Summing up, in the Rich man-Poor man scenario, the equilibrium can be of two types: either the poor agent spends less than if financial constraints would not exist (equilibrium without borrowing) or he risks bankruptcy (equilibrium with borrowing). The first equilibrium occurs when concerns for the future are important and the poor agent is not very poor. The second equilibrium exists in the opposite circumstances and there both agents spend more than in the one shot equilibrium. This result occurs because the rich agent has incentives to force the poor agent to accept a large risk of bankruptcy. Thus, this equilibrium is a predatory type of equilibrium, such as when the US forced the USSR in the early 1980s into a large military expenditure that accelerated the demise of the socialist state. For intermediate values of concern about the future and wealth of the poor agent, an equilibrium might not exist.

4 Poor Man-Poor Man Scenario

In this section we study the case in which no agent can pay the expenses corresponding to the one shot game out of his endowments: $M_a < \bar{G}$ and $M_b < \bar{G}$. For simplicity, we focus here on the case in which both agents have identical endowments, thus, $M_a = M_b = M$. This case is the polar case to that considered in the previous section, where the asymmetry arising from endowments was maximal.

As in the previous section, there are equilibrium without and with borrowing. We start with the former.

In a non borrowing equilibrium, given that the agents are identically constrained, both agents expend their entire resources in the first period. Both agents survive in the second period; therefore, their continuation payoff is identical for both of them and equal to $\delta V/4$. Thus, payoffs are

$$\pi_i^{NB}(M, M) = \frac{1}{2}V - M + \frac{\delta V}{4}, i \in \{a, b\}. \quad [4.1]$$

If agent i deviates and borrows, his continuation payoff changes because he faces bankruptcy with some probability. Thus, by playing $G_i > M$, i 's payoff is

$$\pi_i^B(G_i, M) = \frac{G_i}{G_i + M} V \left(1 + \frac{\delta}{4} \right) - G_i. \quad [4.2]$$

The most profitable deviation will be to play the best reply of i against M , that is $G_i = \sqrt{V(1 + \frac{\delta}{4})}M - M$, giving to agent i a payoff

$$\pi_i^B(G_i, M) = \frac{\sqrt{V(1 + \frac{\delta}{4})M} - M}{\sqrt{V(1 + \frac{\delta}{4})M}} V \left(1 + \frac{\delta}{4}\right) - \sqrt{V \left(1 + \frac{\delta}{4}\right)M} + M. \quad [4.3]$$

An equilibrium with non borrowing will exist iff

$$\pi_i^{NB}(M, M) \geq \pi_i^B(G_i, M), \quad [4.4]$$

The above inequality can be written as

$$\frac{1}{2} - \frac{1}{q} \geq 1 - 2\sqrt{\frac{1}{q} \left(1 + \frac{\delta}{4}\right)} + \frac{1}{q}. \quad [4.5]$$

Thus we have proved:

Proposition 3 *In the Poor Man-Poor Man scenario a non borrowing equilibrium exists iff the following inequality holds*

$$\frac{1}{2} - \frac{1}{q} \geq 1 - 2\sqrt{\frac{1}{q} \left(1 + \frac{\delta}{4}\right)} + \frac{1}{q}.$$

This inequality is pictured in Figure 5 below where the combining values (q, δ) that satisfy eq. [4.5] are those in the area above the dashed line.

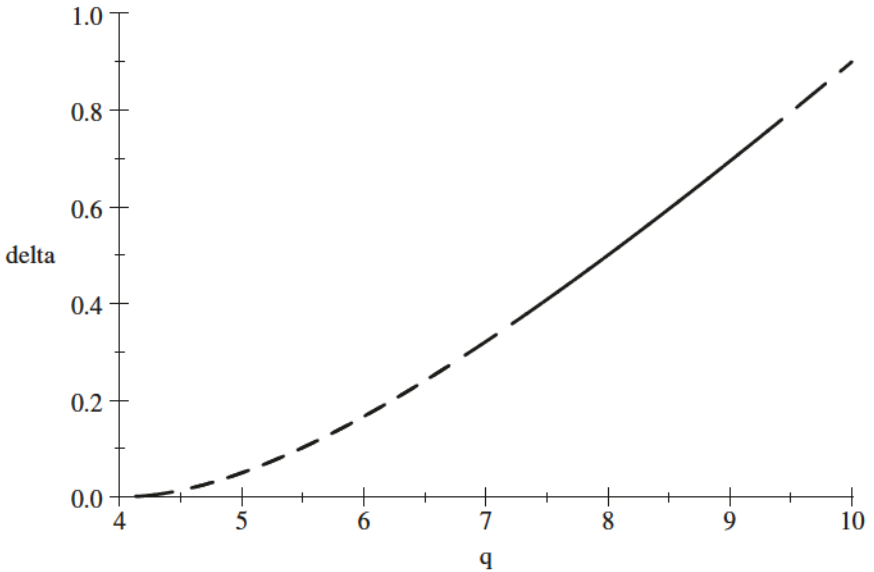


Figure 5: Non borrowing equilibrium in the poor man-poor man scenario.

We can see that, qualitatively, the boundary between the existence and the non existence of a non borrowing is identical to the one in the rich man-poor man scenario.

Now suppose that both agents borrow. In this case both are facing a probability of bankruptcy in the second period. Therefore, with probability p_i agent i will be the only one surviving in the game and his continuation payoff will be $\delta p_i V$. At (G_a, G_b) with $G_i > M$, the payoff for agent i is

$$\pi_i^B(G_a, G_b) = p_i V - G_i + \delta p_i V = p_i(1 + \delta)V - G_i. \quad [4.6]$$

Let (\hat{G}_a, \hat{G}_b) be

$$\hat{G}_a = \hat{G}_b = \hat{G} = \frac{(1 + \delta)V}{4}. \quad [4.7]$$

Let us see that (\hat{G}_a, \hat{G}_b) is an equilibrium with borrowing.

If agent i deviates and does not borrow, that is, plays $G_i \leq M$, he will not face bankruptcy, and with probability $p_i(G_i, \hat{G})$ he will be the only one surviving in the game and obtaining V . With probability $1 - p_i(G_i, \hat{G})$, both agents will survive and agent i will obtain $V/4$. Thus, if he deviates by playing $G_i \leq M$, his payoff will be:

$$\begin{aligned} \pi_i^{NB}(G_i^1, \hat{G}) &= p_i(G_i, \hat{G})V - G_i + \delta(p_i(G_i, \hat{G})V + (1 - p_i(G_i, \hat{G}))V/4) \\ &= p_i(G_i, \hat{G})V \left(1 + \frac{3}{4}\delta\right) - G_i + \delta V/4. \end{aligned} \quad [4.8]$$

Let \tilde{G}_i be the best reply to \hat{G} according to π_i^{NB} without considering the constraints. First, let us see that $\tilde{G}_i > M$. Thus, the best possible deviation would be to play M . Note that $\tilde{G}_i = \sqrt{(1 + \frac{3}{4}\delta)V\hat{G}} - \hat{G}$. Using the value of \hat{G} and that $V/M = q$, we obtain that

$$\sqrt{\left(1 + \frac{3}{4}\delta\right)V\hat{G}} - \hat{G}_b > M \quad [4.9]$$

is equivalent to

$$\sqrt{\left(1 + \frac{3}{4}\delta\right)\frac{(1 + \delta)}{4}} - \frac{(1 + \delta)}{4} > \frac{1}{q}. \quad [4.10]$$

Because $q > 4$, and the left-hand side of eq. [4.10] is increasing in δ , the smallest value of the left-hand side is $1/4$. Thus, condition [4.10] always holds. Therefore, we only need to check that deviating by non borrowing and playing M is not profitable. If agent i deviates and plays M he obtains

$$\pi_i^{NB}(M, \hat{G}) = \frac{M}{\frac{(1+\delta)V}{4} + M} V \left(1 + \frac{3}{4}\delta \right) - M + \delta V/4. \quad [4.11]$$

The deviation is not profitable iff

$$\pi_i^B(\hat{G}, \hat{G}) \geq \pi_i^{NB}(M, \hat{G}). \quad [4.12]$$

That is,

$$\frac{(1+\delta)V}{4} \geq \frac{M}{\frac{(1+\delta)V}{4} + M} V \left(1 + \frac{3}{4}\delta \right) - M + \delta V/4, \quad [4.13]$$

or equivalently,

$$\frac{1}{4} \geq \frac{(1 + \frac{3}{4}\delta)}{\frac{(1+\delta)q}{4} + 1} - \frac{1}{q}. \quad [4.14]$$

Summing up,

Proposition 4 *In the Poor Man-Poor Man scenario a borrowing equilibrium exists iff the following inequality holds.*

$$\frac{1}{4} \geq \frac{(1 + \frac{3}{4}\delta)}{\frac{(1+\delta)q}{4} + 1} - \frac{1}{q}.$$

This inequality is pictured in Figure 6 below where the combining values (q, δ) that satisfy [4.14] are those in the area below the solid line.

Again we see that the qualitative features of the boundary between the pairs of (δ, q) for which a non borrowing equilibrium exists and those for which it does not exists are identical to those in the rich man-poor man scenario.

Finally, we show that there is no equilibrium with only one agent borrowing. If an equilibrium with these characteristics exists, an agent expend their entire resources in the first period and the other borrows and best reply to M . Suppose, without loss of generality, that agent one is borrowing. Let (G_a, M) be such that G_a is the best reply to M taking into account that by borrowing agent one is facing bankruptcy in the second period and agent two is not exposed to bankruptcy, that is, $G_a = \sqrt{V(1 + \frac{\delta}{4})M} - M$. For (G_a, M) to be an equilibrium, agent one should not have incentives to deviate by non borrowing. Given the analysis that we have done in the non borrowing equilibrium (reflected in eq. [4.5]), that deviation will not occur if and only if

$$\frac{1}{2} - \frac{1}{q} \leq 1 - 2\sqrt{\frac{1}{q} \left(1 + \frac{\delta}{4} \right)} + \frac{1}{q}, \text{ with } q = \frac{V}{M}. \quad [4.15]$$

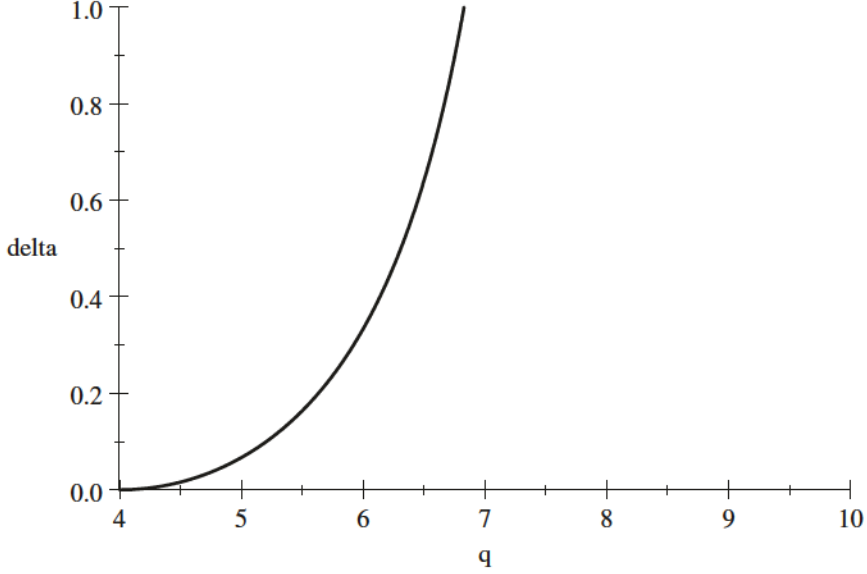


Figure 6: Borrowing equilibrium in the poor man-poor man scenario.

Furthermore, agent two should not have incentives to borrow. If he deviates and borrows the best deviation will be to play the best reply to G_a taking into account that, by borrowing, if he loses the first contest he will be bankrupted in the second period but if he wins he will be the unique contestant in the future. His best deviation is to play $G_b = \sqrt{V(1+\delta)G_a} - G_a$. Agent two will not deviate if and only if $\pi_b^{NB}(G_a, M) \geq \pi_b^B(G_a, G_b)$. That is,

$$p_b(G_a, M)V\left(1 + \frac{3}{4}\delta\right) - M + \delta\frac{V}{4} \geq p_b(G_a, G_b)V(1+\delta) - G_b. \quad [4.16]$$

which rearranging terms and using $q = \frac{V}{M}$ can be written as

$$\frac{\frac{1}{q}\delta}{2\sqrt{(1+\frac{1}{4}\delta)\frac{1}{q}}} \geq \left(1 + \frac{3}{4}\delta\right) - 2\sqrt{(1+\delta)\left(\sqrt{\left(1 + \frac{1}{4}\delta\right)\frac{1}{q}} - \frac{1}{q}\right)}. \quad [4.17]$$

In Figure 7 below we show that an equilibrium with this characteristics never exist. The reason is that being both agents identical, different behaviors are not individually optimal. The area above the solid line corresponds to eq. [4.17] and the area below the dash line corresponds to eq. [4.15]. Thus, eqs [4.17] and [4.15] are not compatible.

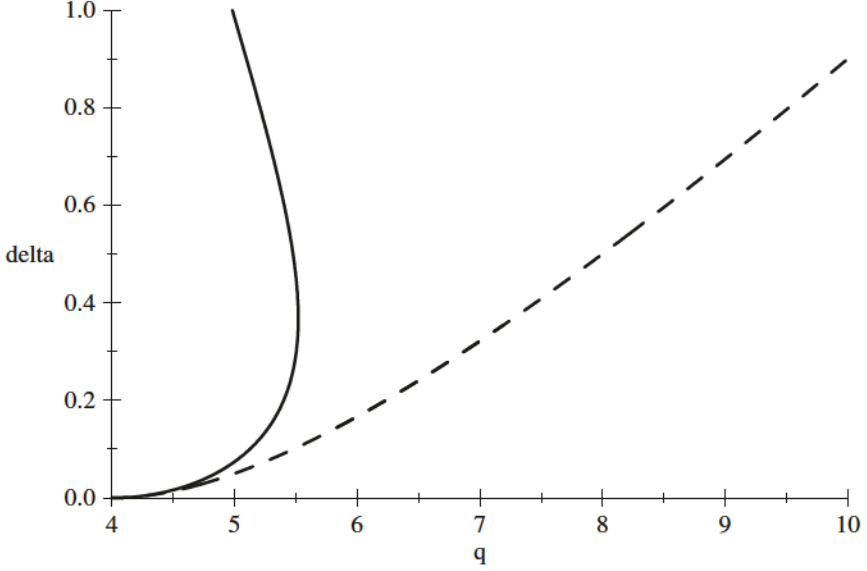


Figure 7: An equilibrium with only one agent borrowing does not exist in the symmetric poor man-poor man scenario.

In Figure 8 we combine Figures 5 and 6. Note that for any pair (q, δ) between the solid line and the dashed both equilibria can coexist; Example 2 below reflects this possibility.

In this scenario, contrarily to what happens in the rich man-poor man scenario, an equilibrium always exists. Moreover equilibrium is not unique: There is a set of values of (δ, q) for which both borrowing and non borrowing equilibria coexists. Thus we see that a more equal endowment causes multiplicity of equilibria and the corresponding coordination problem.

Example 2 *Coexistence of borrowing and non borrowing equilibria.*

Suppose that $V = 80$, $\delta = 1$, $M = 10$ and $q = 8$. Both agents are constrained. Because both agents are identical, their best reply functions are also identical.

$$G_i = \begin{cases} \sqrt{VG_j} - G_j & \text{if } G_j \leq M_j \text{ and } \sqrt{VG_j} - G_j < M_i \\ M_i & \text{if } G_j \leq M_j \text{ and } \sqrt{VG_j} - G_j > M_i, \text{ and } \pi_i^{NB} > \pi_i^B \\ \sqrt{V(1 + \frac{\delta}{4})G_j} - G_j & \text{if } G_j \leq M_j \text{ and } \sqrt{VG_j} - G_j > M_i \text{ and } \pi_i^{NB} < \pi_i^B \\ M_i & \text{if } G_j > M_j \text{ and } \bar{\pi}_i^{NB} > \bar{\pi}_i^B \\ \sqrt{V(1 + \delta)G_j} - G_j & \text{if } G_j > M_j \text{ and } \bar{\pi}_i^{NB} < \bar{\pi}_i^B \end{cases}$$

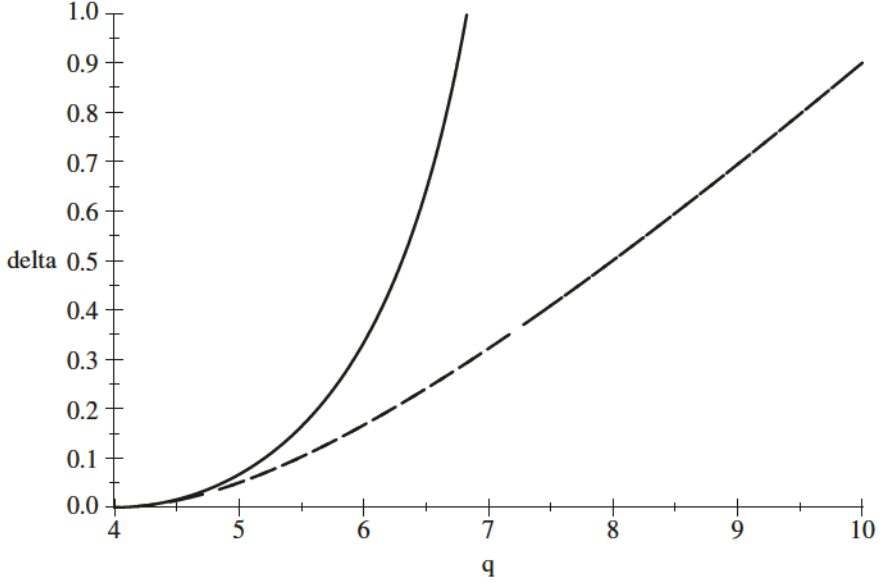


Figure 8: Equilibrium in the poor man-poor man scenario.

where

$$\pi_i^{NB} = \frac{M_i}{M_i + G_j} V - M_i + \delta \frac{V}{4}, \quad [4.18]$$

$$\pi_i^B = \frac{\sqrt{V(1 + \frac{\delta}{4})G_j - G_j}}{\sqrt{V(1 + \frac{\delta}{4})G_j}} V \left(1 + \frac{\delta}{4}\right) - \sqrt{V \left(1 + \frac{\delta}{4}\right) G_j + G_j}, \quad [4.19]$$

$$\bar{\pi}_i^{NB} = \frac{M_i}{M_i + G_j} V(1 + \delta) - M_i, \quad [4.20]$$

$$\bar{\pi}_i^B = \frac{\sqrt{V(1 + \delta)G_j - G_j}}{\sqrt{V(1 + \delta)G_j}} V(1 + \delta) - \sqrt{V(1 + \delta)G_j + G_j}. \quad [4.21]$$

In this example, for any $G_j \in [0, 10]$, $\pi_i^{NB} > \pi_i^B$, and for any $G_j \in (10, 80]$ we have that $\bar{\pi}_i^{NB} < \bar{\pi}_i^B$. The best reply functions of both agents are plotted in Figure 9.

We do have an equilibrium without borrowing in which both agents spend 10 and another equilibrium with borrowing where both agents spend 40. The first equilibrium is not very robust because if, say, player a chooses $10 + \varepsilon$ ($\varepsilon > 0$

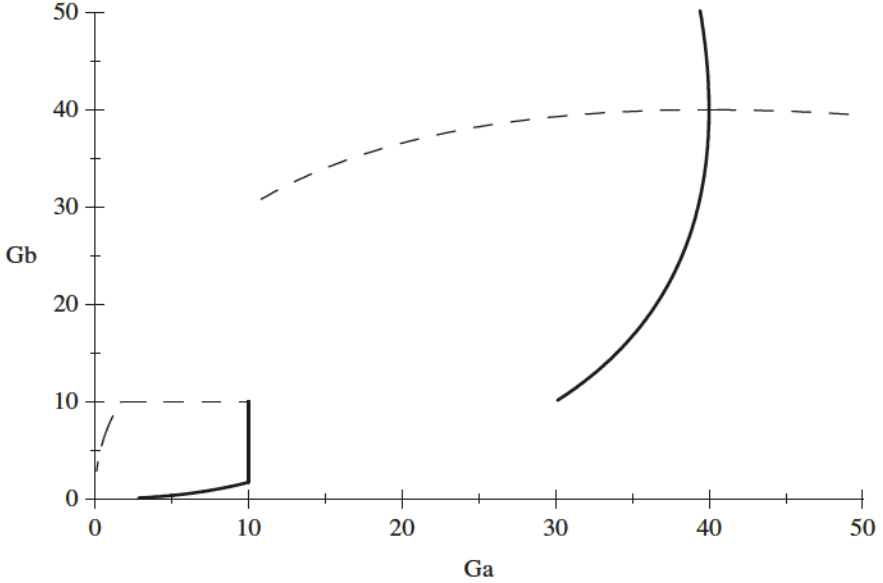


Figure 9: Multiple equilibria. Poor man-poor man scenario.

but still very small) the best reply of player b is far from 10. However, this equilibrium is robust if endowments vary a little.

5 Conclusions and Further Extensions

In this paper, we presented a model of a two-period contest in which agents have monetary endowments and may borrow money. We assume that the inability to repay the loan carries the disappearance of this agent. We have shown that relatively poor agents might take loans. Thus, handicapped agents may take actions that endanger their survival in the long run but that, if successful, substantially reduce the handicap. We have called this the *Despair Effect* and we have shown that it exists in two polar scenarios: Rich Man-Poor Man, in which an agent has unlimited endowments, and Poor Man-Poor Man in which both agents are identical and have relatively small endowments.

Many questions remain to further understand the issues raised in this paper. We present four below.

1. More general CSF.

A more general analysis which do not rely on the form of the CSF is available under request. We show that our qualitative conclusions hold if the CSF yields a best reply which is first increasing and then decreasing.

2. Other scenarios

We also have solved the model when both agents are different but constrained (i.e. they cannot pay their one shot Nash equilibrium expenses with their endowments). Qualitatively, this analysis (available under request) does not bring new insights to the those obtained in the Poor Man-Poor Man scenario.

3. Entry in the second period

In our model, if a poor agent falls into bankruptcy, he leaves the entire field to the other contestant. What if the disappearance of a contestant triggers the entry of another contestant in the second period? In this case, the continuation payoff of the rich agent is always $\delta\bar{\pi}$. If the poor agent borrows in the first period, he faces bankruptcy in the second period; thus his payoff will be

$$\pi_b(G_a, G_b) = p_b(G_a, G_b)V \left(1 + \frac{\delta}{4}\right) - G_b. \quad [5.1]$$

In an equilibrium with borrowing, the expenses in the first period are

$$G_a = \frac{(1 + \frac{\delta}{4})}{(2 + \frac{\delta}{4})^2} V; \quad G_b = \frac{(1 + \frac{\delta}{4})^2}{(2 + \frac{\delta}{4})^2} V, \quad [5.2]$$

Note that G_a is decreasing with δ and G_b is increasing with δ . The payoff for the poor agent is

$$\pi_b^B = \frac{(1 + \frac{\delta}{4})^3}{(2 + \frac{\delta}{4})^2} V. \quad [5.3]$$

Finally, note that the poor agent, once he has the opportunity to borrow, will expend more resources than the rich agent in the first period.⁹ The reason for this result is that, the rich agent does not have an incentive to prey, so it decreases its expenses in the first period. However, equilibrium occurs when actions are strategic substitutes for the poor agent. Thus, the poor agent increases his expenses, and this leads to borrowing. In this case, the shadow effect is not present, and the continuation value of the rich does not change with the risky action of the poor agent. However, the Despair Effect is still present. The poor agent prefers the risky action to compete on

⁹ In this case, an equilibrium with borrowing always exists. A complete characterization of the equilibrium using the Tullock CSF is available on request.

an equal footing in the first period, but because he is facing bankruptcy in the second period, he is even more aggressive in the first period to reduce the probability of bankruptcy. Because equilibrium occurs where actions are strategic complements for the rich, this agent decreases his expenses and the poor increases his expenses, which explains our results above.

4 More Periods

In our model, the world ends in the second period. This implies that there is no strategic concern in the second period. This is odd because it creates an asymmetry between both periods. The way of fixing this problem would be a model with an infinite horizon in which this asymmetry disappears. This model is beyond the scope of the present paper. In any case our model can be interpreted as one in which there is a fixed reward for survival in the second period. In a dynamic model this would correspond to the second part of the value function.

Other important assumptions are the existence of two agents only, a very specific bankruptcy rule in which bankrupted agents disappear and a stylized capital market where any loan can be obtained at the current interest rate. All of these assumptions raise important issues that must be considered in further research. Finally, this paper is totally silent on questions of welfare such as is it optimal to allow bankruptcies? or should contestant firms be taxed/subsidized?

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